

Phase Estimation With Interfering Bose-Condensed Atomic Clouds

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We investigate how to estimate from atom-position measurements the relative phase of two Bose-Einstein condensates released from a double-well potential. We demonstrate that the phase estimation sensitivity via the fit of the average density to the interference pattern is fundamentally bounded by shot noise. This bound can be overcome by estimating the phase from the measurement of \sqrt{N} (or higher) correlation function. The optimal estimation strategy requires the measurement of the N -th order correlation function. We also demonstrate that a second estimation method – based on the detection of the center of mass of the interference pattern – provides sub shot-noise sensitivity. Yet, the implementation of both protocols might be experimentally challenging.

Introduction. The experimental realization of the Bose-Einstein condensation (BEC) opened a new chapter in the field of atom interferometry [1]. The BEC constitutes a bright and well-controllable source of particles, in analogy to the laser light commonly used for interferometric purposes. Atoms are very good candidates for precise measurement of electromagnetic [2–4] or gravitational [5, 6] forces. Moreover, non-classical states have been created via Quantum Nondemolition measurements [7, 8], or exploiting the naturally present two-body interactions [9–11]. These states allow to overcome the limit imposed by the classical physics on measurement precision (shot-noise limit) [12, 13], as recently demonstrated experimentally [14]. An atom interferometer can be implemented using a gas trapped in a double-well potential [15–17], where the two spatial modes localized in its minima play the role of the interferometer’s arms. In the simplest interferometric sequence, first a relative phase θ builds up between the wells and then the BECs are released from the trap and form an interference pattern. In this manuscript we analyze different phase estimation strategies relying on position measurement of the atoms forming this pattern. We show that fitting the average density to the interference pattern [15], gives a sensitivity $\Delta\theta$ bounded by the shot-noise. This limit can be overcome by measuring correlations between the atoms of order not smaller than \sqrt{N} , where N is the total number of atoms. We also demonstrate that the estimation by the N -th order correlation function is *optimal* – the sensitivity saturates the bound set by the Quantum Fisher Information (QFI) [18]. Finally, we present another estimation scheme which can give sub shot-noise sensitivity, based on the measurement of the position of the center of mass of the cloud. Although it involves a probability which is simpler to construct with respect to high-order correlation functions, it requires detection of all N atoms. These results indicate that the achievement of sub shot-noise sensitivity using the interference pattern might prove challenging.

The model. We introduce the two-mode field operator of a bosonic gas in a double-well potential, $\hat{\Psi}(x, t) =$

$\psi_a(x, t)\hat{a} + \psi_b(x, t)\hat{b}$, where $\hat{a}^\dagger/\hat{b}^\dagger$ creates an atom in the left/right well. While the atoms remain trapped, an unknown relative phase θ is imprinted between the wells. This stage is described by a unitary evolution $\hat{U}(\theta) = e^{-i\theta\hat{J}_z}$ of the initial state $|\psi_{in}\rangle$ of the double-well system [19]. After the phase acquisition, the trap is switched off and the initially localized mode functions $\psi_{a/b}(x, t)$ freely expand.

All information which can be extracted from a position measurement is contained in $p_N(\vec{x}_N|\theta)$ [20–22] – the conditional probability of finding N particles at positions $\vec{x}_N = (x_1 \dots x_N)$. It can be expressed in terms of the N -th order correlation function [23] as follows: $p_N(\vec{x}_N|\theta) = \frac{1}{N!}G_N(\vec{x}_N, \theta)$. To evaluate this probability, we write the initial state in the well-population basis, $|\psi_{in}\rangle = \sum_{n=0}^N C_n|n, N-n\rangle$ and suppose that the expansion coefficients are real and possess the symmetry $C_n = C_{N-n}$ [24]. It is convenient to use the Heisenberg representation and evolve the field operator, $\hat{U}^\dagger(\theta)\hat{\Psi}(x, t)\hat{U}(\theta) = \psi_a(x, t)e^{i\frac{\theta}{2}}\hat{a} + \psi_b(x, t)e^{-i\frac{\theta}{2}}\hat{b}$. Then, we expand the Fock states $|n, N-n\rangle$ in the basis of the spin-coherent states [25] and obtain

$$p_N(\vec{x}_N|\theta) = \int_0^{2\pi} \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{d\varphi'}{2\pi} \prod_{i=1}^N u_\theta^*(x_i, \varphi; t) u_\theta(x_i, \varphi'; t) \sum_{n,m=0}^N \frac{C_n C_m \cos[\varphi(\frac{N}{2} - n)] \cos[\varphi'(\frac{N}{2} - m)]}{\sqrt{\binom{N}{n} \binom{N}{m}}}, \quad (1)$$

where $u_\theta(x, \varphi; t) = \psi_a(x, t)e^{\frac{i}{2}(\varphi+\theta)} + \psi_b(x, t)e^{-\frac{i}{2}(\varphi+\theta)}$. In the remaining part of the manuscript, the time t is chosen such that the interference pattern is already formed. The probability (1) is the starting point for the following discussion of various phase estimation strategies.

Estimation via the fit to the density. One of the most natural ways of estimating the value of θ is through the fit of the average density to the interference pattern. The experiment consists in dividing the interference pattern into M bins located at positions x_i and measuring the number of particles n_i in each of them. This procedure is repeated m times: the set

$n_i^{(k)}$, $k = 1, \dots, m$ is recorded and the average $\bar{n}_i = \sum_{k=1}^m n_i^{(k)} / m$ in i -th bin is evaluated. The average occupation $\langle n_i \rangle = \lim_{m \rightarrow \infty} \sum_{k=1}^m n_i^{(k)} / m$ (we assume to be known a priori in the experiment) with free parameter θ is then fitted to the histogram of the measured density $\{x_i, \bar{n}_i\}$, $i = 1, \dots, M$. For small bin size Δx , $\langle n_i \rangle$ is related to the average density by $\langle n_i \rangle = G_1(x_i, \theta) \Delta x$. The value of θ can be determined from the least squares formula $\frac{d}{d\theta} \sum_{i=1}^M \frac{(\bar{n}_i - \langle n_i \rangle)^2}{2\Delta^2 n_i / m} = 0$. The number fluctuations $\Delta^2 n_i = \lim_{m \rightarrow \infty} \sum_{k=1}^m (n_i^{(k)} - \langle n_i \rangle)^2$ (also assumed to be known a priori in the experiment) can be calculated from the probability $p(n_i | \theta)$ of detecting n_i particles in the i -th bin,

$$p(n_i | \theta) = \binom{N}{n_i} \int_{\Delta x_i} d\vec{x}_{n_i} \int_{\mathbb{R} - \Delta x_i} d\vec{x}_{N-n_i} p_N(\vec{x}_N | \theta).$$

Using Eq.(1) we obtain

$$\Delta^2 n_i = G_1(x_i, \theta) \Delta x + [G_2(x_i, x_i, \theta) - G_1^2(x_i, \theta)] (\Delta x)^2. \quad (2)$$

In order to derive the sensitivity $\Delta^2 \theta$ for this estimation method, we notice that, if the number of measurements m is large, according to the central limit theorem the probability distribution for the average \bar{n}_i in the i -th bin tends to the Gaussian $p(\bar{n}_i | \theta) = \frac{1}{\sqrt{2\pi\Delta n_i/\sqrt{m}}} e^{-\frac{(\bar{n}_i - \langle n_i \rangle)^2}{2\Delta^2 n_i / m}}$. Although in every shot the atom counts are correlated between the bins, averaging over large number of experiments washes out this dependence. Therefore, the total probability of measuring the values $\{\bar{n}\} = (\bar{n}_1 \dots \bar{n}_M)$ is a product $P(\{\bar{n}\} | \theta) = \prod_{i=1}^M p(\bar{n}_i | \theta)$. For this Gaussian probability, the condition for the least square fit coincides with the condition for the maximum likelihood estimator (MLE), $\frac{d}{d\theta} P(\{\bar{n}\} | \theta) = 0$. This is a crucial observation since it is well known [26] that, for this choice of the estimator, the sensitivity is given by the Cramer-Rao Lower Bound, $\Delta^2 \theta = F^{-1}$. Here, F is the Fisher information (FI),

$$F = \sum_{\bar{n}_1, \dots, \bar{n}_M=0}^N \frac{1}{P(\{\bar{n}\} | \theta)} \left(\frac{\partial}{\partial \theta} P(\{\bar{n}\} | \theta) \right)^2 \xrightarrow{m \gg 1} m \sum_{i=1}^M \frac{1}{\Delta^2 n_i} \left(\frac{\partial \langle n_i \rangle}{\partial \theta} \right)^2. \quad (3)$$

Since conditions for the MLE and the fit coincide, the sensitivity of the latter is given by the inverse of (3) as well.

We now demonstrate that this sensitivity is bounded by the shot-noise. Let us assume for the moment that the second term in the Eq.(2) – which is proportional to $(\Delta x)^2$ – can be neglected. If this is the case, the particle number distribution is Poissonian and the FI from (3)

simplifies to

$$F = m \sum_{i=1}^M \frac{1}{G_1(x_i, \theta)} \left(\frac{\partial}{\partial \theta} G_1(x_i, \theta) \right)^2 \Delta x \simeq mN \int_{-\infty}^{\infty} dx \frac{1}{p_1(x | \theta)} \left(\frac{\partial}{\partial \theta} p_1(x | \theta) \right)^2, \quad (4)$$

with the one-particle probability $p_1(x | \theta) = \frac{1}{2}(|\psi_a(x, t)|^2 + |\psi_b(x, t)|^2) + \frac{2}{N} \langle \hat{J}_x \rangle \text{Re} [\psi_a^*(x, t) \psi_b(x, t) e^{i\theta}]$. It can be demonstrated [27] that Eq.(4) gives $F \leq mN$, thus $\Delta \theta \geq \Delta \theta_{SN} = \frac{1}{\sqrt{m}} \frac{1}{\sqrt{N}}$ for *any* input state ($\Delta \theta_{SN}$ denotes the shot-noise sensitivity). Below we argue that the inclusion of the second term in the fluctuations does not improve the sensitivity.

In Eq.(2), the first term $G_1(x_i, \theta) \Delta x$ scales linearly with N while the second term, as a function of N , is a polynomial of the order not higher than two, $aN^2 + bN + c$. Since the fluctuations $\Delta^2 n_i$ must be positive, then $a \geq 0$. Otherwise, for large N , no matter how small Δx , we would have $\Delta^2 n_i < 0$. If $a > 0$, the positive second term enlarges the fluctuations and thus worsens the sensitivity. When $a = 0$, the first and second terms in Eq.(2) scale linearly with N , and for small Δx the second term can be neglected, thus we end up again with Eq.(4) (a similar argument can be made in order to conclude that increasing the bin size Δx always worsens the sensitivity with respect to Eq.(4)). This is the first important result of this manuscript: $\Delta \theta_{SN}$ is a lower bound for the fit sensitivity. This is because the maximal value of the FI is expressed in terms of the single-particle probability and does not exploit the correlations between the atoms. Let us now demonstrate that the measurement of these correlations can indeed improve the phase sensitivity.

Estimation via the correlation functions. The estimation method we discuss now relies upon determination of the phase θ from the k -th order correlation function $G_k(\vec{x}_k | \theta)$. We can choose to deduce θ using the MLE [28]: given a k -tuple $\vec{\xi}_k$ of detected atoms' positions, the estimated value of the phase is chosen by maximizing $G_k(\vec{\xi}_k | \theta)$ with respect to θ . The variance $\Delta^2 \theta$ after $m \gg 1$ experiments is given by $\Delta^2 \theta = F_{(k)}^{-1}$, where

$$F_{(k)} = m \frac{N}{k} \int_{-\infty}^{\infty} d\vec{x}_k \frac{1}{p_k(\vec{x}_k | \theta)} \left(\frac{\partial}{\partial \theta} p_k(\vec{x}_k | \theta) \right)^2, \quad (5)$$

with $p_k(\vec{x}_k | \theta) = \frac{(N-k)!}{N!} G_k(\vec{x}_k, \theta)$. The coefficient $\frac{N}{k}$ accounts for the number of independent drawings of k particles from N , i.e. $\binom{N}{k} / \binom{N-1}{k-1}$. We notice that by setting $k = 1$, i.e. the estimator is a single-particle density, we recover the FI from Eq.(4). Therefore, the measurement of positions of N particles independently is, in terms of sensitivity, equivalent to fitting the average density, and thus is limited by the shot-noise.

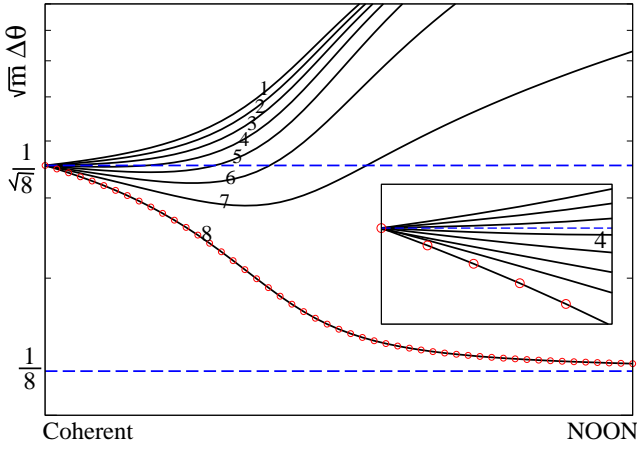


FIG. 1: (color online). The sensitivity $\sqrt{m}\Delta\theta$ (black solid lines) for $N = 8$ calculated by numerical integration of Eq.(5) for various k as a function of $|\psi_{in}\rangle \in \mathcal{A}$. The two limits, $\sqrt{m}\Delta\theta_{SN}$ and $\sqrt{m}\Delta\theta_{HL}$ are denoted by the upper and lower dashed blue lines, respectively. The optimal sensitivity, given by the QFI, is drawn with red open circles. The inset magnifies the vicinity of the spin-coherent state, showing that the sub shot-noise sensitivity is reached starting from $k_{\min} = 4$.

Let us calculate the FI for the case $k = N$. As the interference pattern is formed after a long expansion time, the mode functions can be written as $\psi_{a/b}(x, t) \simeq e^{i\frac{x^2}{2\tilde{\sigma}^2} \mp i\frac{x \cdot x_0}{\tilde{\sigma}^2}} \cdot \tilde{\psi}\left(\frac{x}{\tilde{\sigma}^2}\right)$, where $\tilde{\sigma} = \sqrt{\frac{\hbar t}{m}}$, $\tilde{\psi}$ is a Fourier transform of the initial wave-packets (common for ψ_a and ψ_b) and the separation of the wells is $2x_0$. The above mode functions give the probability (1), which is inserted into Eq.(5). After integration over space we get

$$F_{(k=N)} = m \cdot 4 \sum_{n=0}^N C_n^2 \left(n - \frac{N}{2}\right)^2 = m \cdot 4\Delta^2 \hat{J}_z. \quad (6)$$

This is the second important result of this manuscript: the estimation of θ from the N -th order correlation function is *optimal* – it saturates the QFI, which is a result of maximization of the FI with respect to all possible measurements [18, 29]. According to Eq.(6), all states for which $\Delta^2 \hat{J}_z > \frac{N}{4}$ give $\Delta\theta < \Delta\theta_{SN}$. Looking for sub shot-noise sensitivity, we take a sub-set of these states – a family of ground states of a double-well system with negative interactions, as this is a natural choice in the context of this work. These states range from spin-coherent to the NOON state. For all states in this family, which we denote by \mathcal{A} , the estimation by the measurement of the N -th order correlation function gives $\Delta\theta < \Delta\theta_{SN}$, and reaches the Heisenberg limit $\Delta\theta_{HL} = \frac{1}{\sqrt{m}} \frac{1}{N}$ for the NOON state (as indicated by open circles in Fig. 1).

It is important to know whether it is still possible to achieve sub shot-noise sensitivity by using lower order correlation functions $k < N$. As the space integrals in Eq.(5) for $k \neq N$ cannot be evaluated analytically, we calculate F numerically taking Gaussian wave-packets,

$\tilde{\psi}\left(\frac{x}{\tilde{\sigma}^2}\right) = \left(\frac{2\sigma_0^2}{\pi\tilde{\sigma}^4}\right)^{\frac{1}{4}} e^{-\frac{x^2\sigma_0^2}{\tilde{\sigma}^4}}$ with the initial width of the wave-packets $\sigma_0 = 0.1$ and half of the well separation $x_0 = 1$. In Fig. 1 we plot the sensitivity using Eq.(5) for $N = 8$ atoms as a function of $|\psi_{in}\rangle \in \mathcal{A}$. We observe that the sensitivity improves with growing k and overcomes the shot-noise limit at $k_{\min} = 4$.

Moreover, we numerically checked that, for larger N , k_{\min} tends to \sqrt{N} (more precisely to the closest integer). However, measuring such a (typically) high order correlation function, during the required calibration stage (p_k must be known before performing the phase estimation protocol described above), involves a large experimental effort. Indeed, for every θ , the k_{\min} -dimensional function $p_{k_{\min}}(\vec{x}_{k_{\min}}, \theta)$ must be probed. In the following section, we present a different detection scheme which gives sub shot-noise sensitivity and does not require the knowledge of a multi-dimensional function.

Estimation via the center of mass measurement. Estimation of θ from the measurement of the center of mass requires a relatively simple calibration stage. Positions of N atoms are recorded and from this data location of the center of mass is calculated. Such observable is described by the one-dimensional probability $p_{cm}(x|\theta)$, related to the full N -body probability (1) by

$$p_{cm}(x|\theta) = \int d\vec{x}_N \delta\left(x - \frac{1}{N} \sum_{i=1}^N x_i\right) p_N(\vec{x}_N|\theta),$$

where “ δ ” is the Dirac delta. Modeling the mode-functions by Gaussians, as in previous section, and under the realistic assumption $e^{-x_0^2/\sigma_0^2} \ll 1$, we obtain

$$p_{cm}(x|\theta) \propto e^{-\frac{2x^2\sigma_0^2}{\tilde{\sigma}^4}N} \left[1 + 2C_0^2 \cos\left(N\theta + \frac{2Nx_0}{\tilde{\sigma}^2}x\right)\right].$$

Notice that the above probability depends on θ only for states with non-negligible NOON component, i.e. non-zero C_0 (and thus C_N) [30].

When $p_{cm}(x|\theta)$ is known, the phase can be estimated using the MLE analysis described in the previous section. The sensitivity is given by the inverse of the FI which can be calculated analytically using the definition $F_{cm} = m \int_{-\infty}^{\infty} dx \frac{1}{p_{cm}(x|\theta)} \left(\frac{\partial}{\partial\theta} p_{cm}(x|\theta)\right)^2$. We obtain

$$F_{cm} = mN^2 \left[1 - \sqrt{1 - 2C_0^2}\right]. \quad (7)$$

In Fig.2 we plot the sensitivity calculated by the inverse of the FI (7) as a function of $|\psi_{in}\rangle \in \mathcal{A}$. The estimation through the center of mass is not optimal, but the sensitivity can still be better than shot-noise, and tends to $\Delta\theta_{HL}$ for $|\psi_{in}\rangle \rightarrow \text{NOON}$. We underline that, although the calibration stage is not as difficult as in the case of high-order correlations, phase estimation based on the center of mass measurement requires detection of *all* N

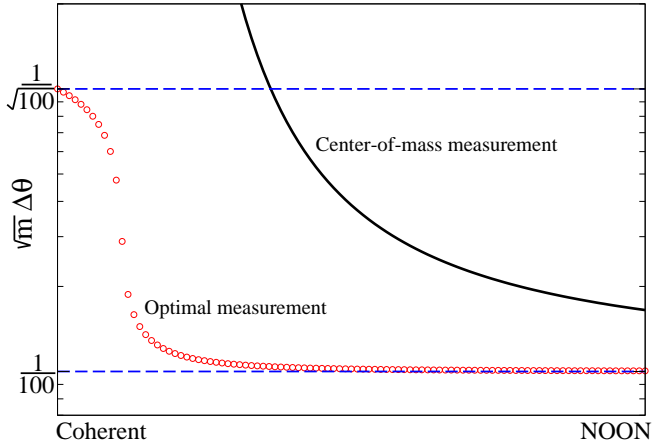


FIG. 2: (color online). The sensitivity $\sqrt{m}\Delta\theta$ (black solid line) for $N = 100$ calculated with Eq.(7) as a function of $|\psi_{in}\rangle \in \mathcal{A}$. The sub shot-noise sensitivity is reached for all states with non-negligible NOON component (see Eq.(7) for details). The values of $\sqrt{m}\Delta\theta_{SN}$ and $\sqrt{m}\Delta\theta_{HL}$ are denoted by the upper and lower dashed blue lines, respectively. The optimal sensitivity, given by the inverse of the QFI, is drawn with the red open circles.

atoms. It can be demonstrated that if just one particle is missed, the sub shot-noise sensitivity is inevitably lost.

Conclusions. We analyzed different phase estimation strategies based on the position measurement of atoms released from a double-well trap. We demonstrated that the fit to the density gives a sensitivity limited by the shot-noise. This bound can be overcome by phase estimation with correlation functions of the order of at least \sqrt{N} , while the N -th order correlation function provides an *optimal* estimation strategy. We also showed that the measurement of the position of the center of mass gives sub shot-noise sensitivity for all states with non-negligible NOON component. Both the protocol involving high-order correlations and the one based on the center of mass position are difficult to implement. The difficulty in achieving sub shot-noise sensitivity can be attributed to the fact that, after formation of the interference pattern, the two spatial modes, corresponding to the arms of the interferometer, cannot be distinguished and the useful information about the correlations between these modes is unavailable. To beat the shot-noise limit, one has to determine the correlations between the particles. These correlations are difficult to extract from the experimental data, thus sub shot-noise sensitivity with two interfering BECs might prove challenging.

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